10/15/19

MIS 7: General Statuses (Continued)

2 Cases:

1) Joint-Life Status $(\sigma = xy)$

T_{xy} = rvr time until the first one death

$T_{xy} = \text{Min}(T_x, T_y)$

2) Last-Survivor Status $(\sigma = \overline{xy})$

$T_{\overline{xy}}$ = rvr time until the last death

$T_{\overline{xy}} = \text{Max}(T_x, T_y)$

Recall: $(\quad)_x + (\quad)_y = (\quad)_{xy} + (\quad)_{\overline{xy}}$

Special Case: $y = \overline{n|} \implies T_{\overline{n|}} = n$

Then $T_{x:\overline{n|}} = \text{Min}(T_x, n) = \begin{cases} T_x & \text{if } T_x < n \\ n & \text{if } T_x \geq n \end{cases}$

Recall: $E[g(X)] = \int_x g(x) \cdot f(x)\,dx$

$\therefore E[T_{x:\overline{n|}}] = \int_0^n t \cdot f_x(t)\,dt + \underbrace{\int_n^\infty n \cdot f_x(t)\,dt}_{= n \cdot {}_nP_x}$

$$\int_0^n t \cdot f_x(t)\,dt$$

IBP

$u = t$ & $v = -{}_tp_x$

$du = dt$ $dv = f_x(t)\,dt$

$$= -t \cdot {}_tp_x \Big|_0^n + \int_0^n {}_tp_x\,dt$$

$$= -n \cdot {}_np_x + \int_0^n {}_tp_x\,dt$$

$$\therefore\ E[T_{x:\overline{n|}}] = \boxed{\begin{aligned} \mathring{e}_{x:\overline{n|}} &= \int_0^n {}_tp_x\,dt \\ \mathring{e}_x &= \int_0^\infty {}_tp_x\,dt \end{aligned}}$$

Likewise
$$E[K_{x:\overline{n|}}] = \boxed{\begin{aligned} e_{x:\overline{n|}} &= \sum_{k=1}^{n} {}_kp_x \\ e_x &= \sum_{k=1}^{\infty} {}_kp_x \end{aligned}}$$

Recursion Formulas:

Recall $e_x = p_x \cdot (1 + e_{x+1})$

(1-year)
$$e_{x:\overline{n|}} = p_x \cdot (1 + e_{x+1:\overline{n-1|}})$$

(2-year):
$$e_{x:\overline{10|}} = \underbrace{p_x + {}_2p_x} + {}_2p_x \cdot (p_{x+2} + {}_2p_{x+2} + \dots + {}_8p_{x+2})$$

$$= e_{x:\overline{2|}} + {}_2p_x \cdot e_{x+2:\overline{8|}}$$

This type of recursion works in the continuous case, too.

Examples:

$$\mathring{e}_{50:\overline{25}|} = \mathring{e}_{50:\overline{15}|} + {}_{15}p_{50} \cdot \mathring{e}_{65:\overline{10}|}$$

$$e_{47:\overline{12}|} = e_{47:\overline{3}|} + {}_{3}p_{47} \cdot e_{50:\overline{9}|}$$

MISM Exercises (Video Solution)

4) Suppose mortality is DML $(\omega = 120)$; $x = 50$

$$\therefore \quad {}_tp_{50} = \frac{70-t}{70}$$

Q: $\mathring{e}_{50:\overline{10}|} = ?$

A: $$\mathring{e}_{50:\overline{10}|} = \int_0^{10} {}_tp_{50}\,dt = \int_0^{10} \left(\frac{70-t}{70}\right) dt$$

$$= \frac{1}{2}\left(\frac{70-t}{70}\right)^2 (+70) \Bigg|_{\cancel{0}\,10}^{\cancel{10}\,0} = \frac{70}{2} - \frac{70}{2}\left(\frac{6}{7}\right)^2$$

$$= 35\left(1 - \frac{36}{49}\right) = 35 \cdot \frac{13}{49} = \frac{65}{7}$$

Alternatively, for DML(ω) models,

$$\mathring{e}_{x:\overline{n}|} = n \cdot {}_np_x + \frac{n}{2} \cdot {}_nq_x$$

Here, $$\mathring{e}_{50:\overline{10}|} = 10 \cdot {}_{10}p_{50} + 5 \cdot {}_{10}q_{50}$$

$$= 10 \cdot \frac{60}{70} + 5 \cdot \frac{10}{70} = \frac{65}{7}$$